MATHEMATICAL DESCRIPTION OF PROCESSES GOVERNING THE OPERATION OF MULTISTAGE AIR-COUNTERFLOW EQUIPMENT

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A mathematical description is given of the stochastic nature of the process of transfer of material from one reactor to another in multistage air-counterflow equipment. A modeling technique has been developed for experimental determination of the basic characteristics of the process.

One of the basic features of multistage air-counterflow equipment [1–3] is the stochastic nature of the residence time of the material being processed in the reactors. There are certain characteristics of this random process which permit the relationship between the main thermal and design parameters of such equipment to be uniquely established. In fact, it is possible to choose these parameters in such a way as to secure reliable, stable operation—in the thermal and hydrodynamic sense—and to attain the required degree of processing of the material.

Analysis of various regimes of operation of air-counterflow systems indicates that the most effective technique is to use dispersed streams, intermediate between fluids and gas suspensions [1].

The physical nature of the effect involved in this process is as follows.

In the axial part of the stream, near the throat, the aerodynamic force acting on a particle is decisive and therefore in this part of the diffuser the particle acquires a velocity which coincides in direction with the stream velocity. At a certain height, however, the weight of the particle begins to predominate, and the motion of the particle is slowed down. In the converging section the aerodynamic force of the stream again increases, and, at a certain critical level the resultant force is again directed upwards. If the particle attains the critical level, then it is carried over from the reactor in question to the previous one. If the vertical component of the particle velocity becomes equal to zero below the critical level, the lateral turbulent fluctuations throw the particle into the peripheral, low-velocity part of the stream, where the main process is a fall under gravity until the particles enter, the throat region. It may then happen that, because of the turbulent fluctuations of gas velocity in the throat region, the aerodynamic resistance of the stream cannot slow the particle down, and it passes into the next reactor. Otherwise the whole cycle is repeated.

We will examine an equipment consisting in the general case of n reactors (see figure).

Assuming that we are operating with a polyfractional material, we will describe a particle fraction by its weight q (or by the diameter d of the equivalent sphere) and introduce the function $f_k(q, t)$ to determine the quantity (weight) and the polyfractional particle distribution in the k-th reactor.

To be exact, $f_k(q, t)\Delta q$ is the number of particles with weight in the range from q to $q + \Delta q$ located at time t in the reactor in question.

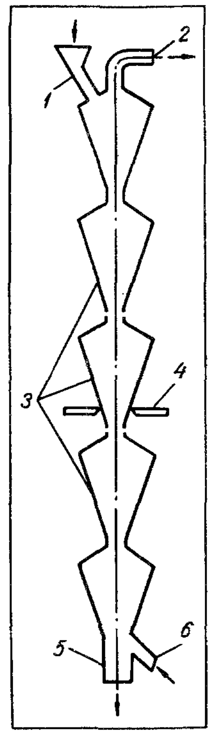

Diagram of air-counterflow equipment: 1) raw material input; 2) flue gas outlet; 3) working reactors; 4) burners; 5) finished product output; 6) air supply.

If we neglect the transformations which the particles undergo during processing in the reactor, we may obtain the following expression for the rate of change of the function $f_k(q, t)$:

$$\frac{\partial f_k(q,\ t)}{\partial t} = a_{k-1} f_{k-1}(q, t) + c_{k+1} f_{k+1}(q, t) - (a_k + c_k) f_k(q, t). \quad (1)$$

We assume that the average number of particles, of weight q, entering the (k + 1)-th reactor (ejected into the (k − 1)-th reactor) during an infinitely small time Δt is proportional to this time and to the current value of $f_k(q, t)$.

In other words, the product $a_k(q)\Delta t(c_k(q)\Delta t)$ gives the probability of escape of a particle q into the (k + 1)-th reactor (ejection into the (k − 1)-th reactor) during the infinitely small time Δt.

Neglecting the ejection of particles into the preceding reactors (when the motion is directed this effect is insignificant), we obtain the following relations for the passage of particles from reactor to reactor:

$$\frac{\partial f_k(q,\ t)}{\partial t} = a_{k-1}f_{k-1}(q,\ t) - a_k f_k(q,\ t);\ k = 1, 2, \ldots, n; \quad (2)$$

where $a_0 f_0(q, t) = F_0(q, t)$ determines the fractional charge distribution.

Assuming that with $t = 0$, $f_k(q, 0) = 0$ $(k = 1, 2, \ldots, n)$, the solution of Eq. (2) will be

$$f_1(q,\ t) = \int_0^t F_0(q,\ t')\exp[-a_1(t-t')]\,dt',$$

$$f_k(q,\ t) = a_1 a_2 \ldots a_{k-1}\int_0^t F_0(q,\ t') \times$$

$$\times \sum_{i=1}^{k} \frac{\exp[-a_i(t-t')]}{\prod_{\substack{s=1\\ s\neq i}}^{k}(a_s - a_i)}\,dt',$$

$$k = 2,\ 3,\ \ldots,\ n. \quad (3)$$

In the special case when the loading of materials into the equipment is intermittent $(F_0(q, t) = F^*(q)\delta(t))$, the solution (3) takes the form

$$f_1(q,\ t) = F^*(q)\exp(-a_1 t),$$

$$f_k(q,\ t) = a_1 a_2 \ldots a_{k-1}F^*(q)\sum_{i=1}^{k}\frac{\exp(-a_i t)}{\prod_{\substack{s=1\\ s\neq i}}^{k}(a_s - s_i)},$$

$$k = 2,\ 3,\ \ldots,\ n. \quad (4)$$

For a steady regime with a constant input rate we have

$$f_k(q) = F_0(q)/a_k, \quad k = 1,\ 2,\ \ldots,\ n. \quad (5)$$

The quantities examined allow a quite simple determination of a number of parameters describing air-counterflow systems.

With the help of these quantities, we may determine limits of variation of the raw material input rate that permit stable operation of the equipment (depending on the flow rate capacity of the throats). Thus, in the case of steady operation of the equipment, it is easy to obtain

$$F \leqslant \beta\rho g V_k \bigg/ \int_0^{q_0} q\frac{\sigma(q)}{a_k(q)}\,dq,$$

$$Q = F\int_0^{q_0} q\,\sigma(q)\,dq.$$

Here, $\beta = \int_0^{q_0} q f_k(q)\,dq/\rho g V_k$ is the volume concentration of solids in the reactor, which usually characterizes the various regimes of operation of systems with through disperse streams [1]; and $\sigma(q)$ is the fractional distribution of the charge material.

It follows from Eq. (1) that the probability density of the stochastic quantity—the residence time of particles in the reactors—is equal to

$$p_k(\tau) = a_k\exp(-a_k\tau). \quad (6)$$

Hence, in particular, we obtain the result that the quantities a_k are inversely proportional to the mean particle residence time in the reactor, since

$$\tau_m = \int_0^\infty \tau\,p_k(\tau)\,d\tau = \frac{1}{a_k}. \quad (7)$$

Using Eq. (5), Eq. (6) may be transformed to the form

$$p_k(\tau) = \frac{F_0}{f_k}\exp\left(-\frac{F_0}{f_k}\tau\right),$$

i.e., in the particular case when there is steady operation of the equipment and the values f_k are known, Eq. (6) coincides with the expression for probability density of the dwell time of particles in the reactors, as obtained in fluidization and suspended state theory [4–6].

In a number of cases, knowledge of the function (6) allows us to determine all the changes which the material undergoes during processing, i.e., to describe the system completely [6].

The equipment examined is designed for heat treatment of the raw material. The determination of the temperature to which the particles are heated reduces to integration of the usual equation of external heat transfer (the processing of particles in the heating of which internal thermal resistance plays a substantial part is scarcely efficient in the equipment examined):

$$c(T_i)\rho v\frac{dT_i}{dt} = \alpha S(u_k - T_i). \quad (8)$$

One possible means of taking account of the stochastic nature of the residence time is to apply Monte Carlo methods [7].

By replacing Eq. (8) by its finite-difference analog, and using the a_k characteristics, we may determine at each time step whether a particle under examination has left the reactor or not. In fact, by using a computer to solve this problem and obtaining, by some means or other, a sequence of pseudo-random numbers Ψ_n, uniformly distributed in the range (0, 1), we calculate that a particle will leave a reactor, if $0 \leq \Psi_n < a_k\Delta f$, and will remain in it, if $a_k\Delta t \leq \Psi_n \leq 1$.

This process allows us to track a particle up to the time that it leaves the equipment and to determine the temperature that it acquires.

The temperature to which the particles have been heated allows us to assess what physical and chemical transformations have occurred in the material being processed, and, therefore, to assess the quality of the process.

By repeating this kind of process many times and by taking the arithmetic mean, we may obtain the mathematical expectation (the mean) of the desired quantities, as well as the variance and other probability characteristics.

The quantities a_k may be determined quite simply by experiments on models. The experiment may be carried out both in the steady-state regime of operation of the model and with the intermittent input of raw material. In the first case the quantities a_k are determined directly from Eq. (5), while in the second case approximate of graphical methods may be applied to solve Eq. (4) with respect to a_k.

We will derive a similarity criterion for modeling from the probable value of the product $a_k \Delta t$.

The probability that a particle will drop may be represented as

$$a_k \Delta t = P_1 P_2,$$

where P_1 is the probability of the particle appearing in the throat region during a time Δt (for small Δt, $\Delta t\ P_1 = \Delta t / t_0$, where t_0 is the time during which a particle moves in an "orbit" inside the reactor); P_2 is the probability of a particle dropping under the conditions existing in the throat region.

We obtain an expression for the probability P_1, by processing the appropriate transformed equations of motion of a particle in the reactor, under a self-similar regime of operation (usually observed in equipment of this kind):

$$P_1 = \frac{\Delta t \sqrt{g/r_k}}{\varphi_1(\mathrm{tg}\,\gamma_1;\ \mathrm{tg}\,\gamma_2;\ h_1/r_k;\ \mathrm{Fr};\ B;\ C)}. \qquad (9)$$

The probability P_2 is determined solely by the relation between the forces acting on the particle at the time at which it is located in the throat, i.e., the quantities $\sim d^3\ (\rho - \rho_0')$ and $\sim d^2 \xi \rho' w^2$.

Here w is the instantaneous velocity of the stream at the point under examination (the absolute velocity of a particle in the throat region is very small).

The velocity w is a stochastic quantity, the distribution of which may be expressed in the general case as a function of w_m—mean velocity in the throat [8].

The particle may drop when $w \leq w_{cr}$, where w_{cr} is determined from the relation

$$4 d \rho g = 3 \xi \rho_0 w_{cr}^2.$$

Then

$$P_2 = P(w \leq w_{cr}) = \varphi_2(\mathrm{Fr};\ B).$$

Finally, we obtain

$$A \equiv a_k \sqrt{r_k g} = \Phi(\mathrm{tg}\,\gamma_1;\ \mathrm{tg}\,\gamma_2;\ h_1/r_k;\ \mathrm{Fr};\ B;\ C). \qquad (10)$$

In deriving Eqs. (9) and (10), it was assumed, for definiteness, that the gas density varies over the height of the reactor according to a linear law:

$$\rho'(s) = \rho_0 + bs, \quad 0 \leqslant s \leqslant h.$$

The criterial Eq. (10) is simplified considerably if we neglect the influence of this change of gas density on the hydrodynamics of the process, while retaining the geometric similiarty of shape of the model and the working equipment. We then obtain

$$A = \Phi_1(\mathrm{Fr};\ B).$$

From model experiments the following formula* was obtained for the determination of the parameter $A = a\sqrt{hg}$:

$$A = 10^{9.8}\,\mathrm{Fr}^{-5.5} B^{0.5}$$

$$\text{for } 100 < \mathrm{Fr} < 250; \quad 10 < B < 35. \qquad (11)$$

Here a certain difference in the reactors proved to be important. In Eq. (11) we took the reactor height h as a characteristic dimension; the characteristic velocity is the mean velocity of the stream in the corresponding throat.

The residence time of particles in the reactor, as calculated using Eq. (11), is in satisfactory agreement with the value determined experimentally on a hot model of the equipment.

The equipment investigated was designed for processing clay to form fireclay. The quality of the firing of the particles is characterized by their water absorption. By determining the water absorption as a function of the temperature to which the particles were heated [9], which was found according to the method described, we also obtained satisfactory agreement between the calculated and the experimentally determined quantities both as regards the quality of fire-clay obtained, and as regards the velocities and temperatures present to secure this quality.

The above approach to the study of operation of equipment of the air-counterflow type may be used for investigating other systems with through disperse streams (for example, equipment with opposing jets [10]), in which a substantial role is played by the stochastic nature of the residence time of the material being processed in the reactors of the system.

*For this equipment the maximum permissible value was $\beta \sim 0.035$.

NOTATION

q is the weight characteristic of particle fraction; q_0 is the weight of maximum fraction; n is the number of reactors; t, τ is the time; τ_m is the mean time; a_k c_k are the characteristics describing escape and ejection of particles; d, ρ, v, S, T are the diameter, material density, volume, surface area, and temperature of particles; V is the reactor volume; F_0 is the fractional input rate; F is the total (particle number) input rate; Q is the mass input rate; u, c, $\bar{\rho}'_0$ is the temperature, specific heat, and density of gas; w, w_m, w_{cr} are the instantaneous, mean, and critical velocity of stream; α is the heat transfer coefficient; γ_1, γ_2 are the opening angles of the divergent and convergent sections of the reactors; r is the throat radius; h is the reactor height; h_1 is the height of one of the cones making up the reactor; ξ is the resistance coefficient; g is the acceleration due to gravity; β is the volume concentration; s is the height coordinate of reactor; $\mathrm{Fr} = w^2/gr$, $B = \rho d/\rho'_0 r$, $C = br/\rho'_0$, $A = a\,(r/g)^{1/2}$ are similarity criteria; b is a coefficient. Subscripts: k refers to the reactor, i to the particle.

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